



2: Radiation interaction with matter

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Photon interactions

Type of Interaction	Effect of interaction
1) Atomic electron	a) Total absorption
2) Nucleus	b) Elastic scattering (coherent)
3) Electric field of the nuclei and atomic electrons	c) Inelastic scattering (incoherent)

Among the possible interaction processes, only 5 of interest in radiological physics:

- 1. (1b) atomic electrons/coherent scattering \rightarrow Thomson/Rayleigh scattering
- 2. (1a) atomic electrons/total absorption \rightarrow photoelectric effect
- 3. (1c) atomic electrons/incoherent scattering \rightarrow Compton scattering
- 4. (3a) electric field/total absorption \rightarrow pair/triplet production
- 5. (2a) nucleus/total absorption \rightarrow photonuclear interaction

Seems familiar? Yes, this is taken (stolen actually) from prof. Veronese's slides. This and other following slides for a "fast" recap Questions: What are the energy range for photoelectric, Compton and pair production interactions?

Photon interactions

The relative importance of photoelectric effect, Compton effect and pair production depends both on the **photon energy** and the **atomic number** of the absorbing medium.

- Photoelectric effect is predominant at lower energies
- Compton effect is predominant at medium energies
- Pair production is predominant at higher energies
- For low-Z media (carbon, air, water, human tissue) the region of Compton effect dominance is very broad (from ~ 20 keV to ~ 30 MeV)
- This window gradually narrows with increasing Z



Photon attenuation coefficient



1) very narrow photon beam

2) thin attenuator (small number of interaction events)

3) small detector and far away from the attenuator \rightarrow The only photons that we want to measure are those that **undergo no interaction**

The number of photons (n) which interact in the attenuator is proportional to the number of incident photons (N) and the thickness of the attenuator (Δx) .

The constant of proportionality $\boldsymbol{\mu}$ is the linear attenuation coefficient

Photon attenuation coefficient

 $n = \mu \cdot N \cdot \Delta x$ can be rewritten as:

where ΔN is the variation of the number of photons $\Delta N = -\mu \cdot N \cdot \Delta x$ passing through the attenuator (negative since we are reducing the number of photons in the beam) \rightarrow $\Delta N = (N - n) - N = -n$

Integrating the equation we get the simple **exponential attenuation law**:

 $N = N_0 \cdot e^{-\mu \cdot x}$ To be used to calculate the attenuation on any thickness of material



The depth where the beam has been reduced by a factor 1/e is the mean free path (λ) of the photon:

 $\lambda = 1/\mu = \langle x \rangle$



Photon cross section



Heavy charged particles interaction

For a given energy, the macroscopic cross section for an electron is several order of magnitude higher than a photon.

Above 1 MeV, electrons have a MFP between collisions of the order of 10⁻⁵-10⁻⁶ cm, whereas it is tens of centimetres for a photon in the same medium.

A charged particle interacts with nearly every atoms along its path, losing energy each time in atomic excitation ad ionization. The vast majority of inelastic interactions transfer only a minute fraction of the kinetic energy of the incident particle → continuous slowing down approximation (CSDA).

The main changes in the direction of the incident particle is due to elastic interactions with the charged atomic nucleus.



Heavy charged particle interactions



- Inelastic hard collision with the atom the atom is ionized and the knock-out electron can be quite energetic (δ rays)
- Inelastic soft collision with the atom the electron is promoted to an excited level in the atom
- Inelastic interaction with nucleus: it may result in the fragmentation of one or both the particles, with the emission of Bremsstrahlung X-ray photons
- Elastic interaction with nucleus: the projectile is scattered by the nucleus without losing kinetic energy

Bethe-Bloch formula

$$-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln \left(\frac{2m_e \gamma^2 v^2 W_{max}}{I^2} \right) - 2\beta^2 - \delta - 2\frac{C}{Z} \right]$$
(1.2)

$$r_e =$$
 classical electron radius

$$m_e =$$
 electron mass

$$N_a =$$
 Avogadro constant

- Z = atomic number of absorbing material
- A = atomic weight of absorbing material
- $\rho =$ density of absorbing material
- z = charge of incident particle
- $\delta = density correction relevant at high energies$
- C = shell correction relevant at low energies
- $W_{max} = \max_{\text{for a incident particle with mass M [8]}}$
- m_{max} for a incident particle with mass M [8]

I= mean excitation potential
$$[\mathfrak{P}] = \begin{cases} 12Z+7, & \text{eV if } Z \le 13\\ 9.76Z+58.8Z^{-0.19} & \text{eV if } Z > 13 \end{cases}$$

$$=\frac{2m_ec^2\beta^2\gamma/2}{1+2\gamma m_e/M+(me/M)^2}$$
eV if $Z\leq 13$

• projectile with $0.1 \le \beta \gamma \le 1000$ Maximum kinetic energy in PT: -protons ~250 MeV -12C ~ 450 MeV/u $\beta \sim 0.6$ and $\beta \sim 0.7$

- materials with intermediate charge
- projectile: dE/dx~z²/β²
- Target: dE/dx~ ρ · ln(1/l²)

Mean excitation potential

- It is the characteristic of the target only without any dependence on the projectile
- It describes how easily a target, (e.g.: molecule or atom) can absorb kinetic energy from a projectile, primarily as electronic (including ionization) and vibrational (including fragmentation) excitation
- In an atom there are several ionization/excitation levels to be considered and the mean excitation energy can be defined as a logarithmic average of all the possible atomic levels
 In I = ∑f i ln(Ii)

 Please approximation: I (a) ()=10*7: reasonable if 7>20
- Bloch approximation: I (eV)=10*Z; reasonable if Z>20
- A possible better approximation [0]: $I = \begin{cases} 12Z+7, & eV \text{ if } Z \leq 13\\ 9.76Z+58.8Z^{-0.19} & eV \text{ if } Z > 13 \end{cases}$
- it can be consistently determined theoretically, but the experimental measurements are difficult to perform and they can lead to distinct results.
- Typically, the mean excitation potential is evaluated from the stopping power or the range measurements, but the results depend considerably on the initial assumptions made concerning the method of extraction and on the projectile properties
- Example: I of water ranges 74.6 81.8 eV with an average of $79.2 \pm 1.6 \text{ eV}$

 $\sum f_i = 1$

Corrections to Bethe-Bloch

Bethe-Bloch approach: reliability

- Good agreement with the NIST data (see later) in the intermediate and high energy regions.
- Discrepancy at low kinetic energy



Further effects should be considered:

- Shell correction → low energy
- Barkas and Block correction → low energy
- **Density effect** (polarization effect) → **high energy**

Bethe-Bloch shell correction

Bethe- Bloch approach: shell correction

Bethe's approach is still based on the Born approximation and assumes that the velocity of the heavy charged particle is much larger than the velocity of the bound orbital electrons of the absorber.

At low kinetic energy this assumption fails: **orbital electrons stop participating in energy transfer from the charged particle when their velocity becomes comparable to the charged particle velocity.**

Ignoring this effect leads to an overestimation of both the *I*-value and the atomic number of the absorber, resulting in an **underestimation of the mass collision stopping power** calculated with the Bethe-Bloch formula (note that the errors in *I* and Z act in opposite directions).

The so-called shell correction term C/Z is introduced to account phenomena. The **correction term C/Z** is a **function of the absorbing medium and charged particle velocity**; however, for the same medium and particle velocity, it is the same for all particles including electrons and positrons.

Electronic stopping power



Bragg curve



 A particle traveling through a material loses its speed and dE/dx increases

At the end of the particle trajectory:

- the particle velocity is comparable to the orbital velocity of the bounded electrons and the shell correction (C/Z) comes into play
- In the low kinetic energy region (~10 MeV/u for light ions) the stopping power reaches a maximum in the Bragg peak.
- After the maximum energy loss, the sum of ionization and recombination processes reduces the effective charge of the incident particle, lowering its energy loss

$$Z_{eff} = Z[1 - \exp(-125\,\beta\,Z^{-2/3})]$$

Nuclear stopping power

At very low energies (<10 KeV/u), interactions with atomic nuclei start to become relevant



Energy straggling

Since the stopping power is a mean value, a large number of collisions in the slowing down process can cause a broadening of the Bragg peak, leading to the energy and range straggling effect

- Energy losses of massive charged particles are a statistical phenomenon
- In each interaction, different amount of energy can be transferred to atomic electrons



For thin layer or low density materials

- There are few collisions, some with high energy transfer
- the energy loss distribution is asymmetric with a tail at high energy (landau tail)

Energy straggling



- Energy losses of massive charged particles is a statistical phenomenon
- In each interaction, different amount of energy can be transferred to atomic electrons
- The energy loss can be parametrized by the Landau function

$$f(\lambda) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(\lambda + e^{-\lambda})\right)$$

with $\lambda = \frac{\Delta E - \overline{\Delta E}}{C\frac{m_e c^2}{\beta^2} \frac{Zz}{A} \rho \Delta x}$

Energy straggling

- For thick layer or high density materials
- There are many collisions
- The energy loss distribution is Gaussian due to the central limit theorem

$$f(\Delta E) = \frac{1}{\sqrt{2\pi\sigma_E}} e^{\frac{(\Delta E - \langle \Delta E \rangle)^2}{2\sigma_E^2}}$$

• The width of the Gaussian depends both on the material and on the projectile

$$\sigma_E = 4 \pi Z_{eff} Z e^4 N_A \Delta x \left(\frac{1 - \beta^2/2}{1 - \beta^2}\right)$$

Particle range

Range of Charged Particles

The **range** of a charged particle of a given energy and type in a given medium is the expectation value of the path length that it follows until it comes to rest

Experimental concept: the range R of a charged particle in a particular absorbing medium is the thickness of an absorber that the particle can just penetrate

Heavy charged particle Light charged particle $\overline{R} \cong R_{CSDA}$ Darticle of an $\overline{R} < R_{CSDA}$

- Heavy charged particles: do not experience radiation losses, transfer only small amounts of energy in individual ionizing collisions, and mainly suffer small angle deflections in elastic collisions. Their path through an absorbing medium is thus essentially rectilinear.
- Light charged particles: can lose significant energy in individual ionizing collisions and individual radiation collisions. They can also be scattered with very large scattering angles, their path through the absorbing medium is very tortuous.

Particle range

Range of Charged Particles

Several definitions of range according to the particles and applications (e.g. projected range, CSDA range, maximum range, the 50% range, practical range, the therapeutic range).

CSDA range (continuous slowing down range)

The CSDA range of a charged particle with initial kinetic energy E_0 :

 $R_{CDSA}(E_0) = \rho \int_0^{E_0} \frac{dE}{S_{col}(E) + S_{rad}(E)} = \rho \int_0^{E_0} \frac{dE}{S_{tot}(E)} \qquad [g \cdot cm^{-2}]$



The CSDA range is a **calculated quantity** that represents the mean path length along the particle's trajectory and not necessarily the depth of penetration in a defined direction in the absorbing medium.

For heavy charged particles, R_{CSDA} is a very good approximation to the average range of the charged particle in the absorbing medium, because of the essentially rectilinear path of the charged particle in the absorbing medium; for light charged particles the CSDA range can be up to twice the average range R

Range scaling laws

The energy loss of charge particles can be translated in a maximum range R (different from photons: attenuation)

$$E_0 = \int_0^R \frac{dE}{dx} dx \qquad \frac{dE}{dx} \propto \frac{z^2}{\beta^2} \ln\left(\frac{2\gamma^2 \beta^2 m_e c^2}{I_0} + ...\right) = z^2 f_\beta(\beta) = z^2 f_E(E)$$

The range R can be written as

$$R = \int_{0}^{R} dx = \int_{0}^{E_{0} - mc^{2}} \frac{dE}{z^{2} f_{E}(E)} = \frac{mc^{2}}{z^{2}} \int_{0}^{\beta_{0}} \frac{\beta d\beta}{f_{\beta} (1 - \beta^{2})^{\frac{3}{2}}} \quad dE = \frac{mc^{2}\beta d\beta}{(1 - \beta^{2})^{\frac{3}{2}}}$$

Some useful scaling laws and behaviour can be obtained:

 $\frac{R_a(\beta)}{R_b(\beta)} = \frac{m_a z_b^2}{m_b z_a^2}$

$$\frac{R_1}{R_2} = \frac{\rho_2}{\rho_1} \frac{\sqrt{A_1}}{\sqrt{A_2}}$$

$$R = \int_0^{E_0} \frac{1}{\frac{dE}{dx}} dE \propto E_0^2$$

Exercise: Try to deduce the range scaling law exploiting the CSDA approximation

Example of particle range



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Range straggling



The dE/dx is a stochastic process and fluctuations of dE/dx and range values are observed experimentally

Range straggling



Figure 2.4 Plots of energy distribution of a beam of initially monoenergetic charged particles at various penetration distances. E is the particle energy and X is the distance along the track. (From Wilken and Fritz.³)

The dE/dx is a stochastic process and fluctuations of dE/dx and range values are observed experimentally

• Larger dE/dx lead to smaller fluctuations

Range straggling



Figure 1.2: Bragg peak profiles of a 200 MeV proton beam in water calculated with a change in the mean ionization potential of $\pm 2.5\%$ [18].



The dE/dx is a stochastic process and fluctuations of dE/dx and range values are observed experimentally

- Larger dE/dx lead to smaller fluctuations
- the mean excitation potential (I) is the main source of uncertainty for the stopping power and the range evaluation
- on a proton beam of 200 MeV, varying the mean excitation potential with the values found in literature, the beam range fluctuates of millimeters.
- The range straggling dependence on mass:

$$\frac{\sigma_r}{R} = f\left(1 - \gamma\right) \frac{1}{\sqrt{M}}$$

Multiple Coulomb Scattering (MCS)

In addition to the range straggling that occurs along the incident particle direction, the elastic Coulomb scatterings between the projectile and the target material nuclei lead to a lateral deviation of the incident particle direction

• The elastic Coulomb scattering is described by the Rutherford cross section formula

$$\frac{d\sigma_R}{d\Omega} = \left(\frac{zZe^2}{pv}\right)^2 \frac{1}{4\sin^4\left(\frac{\vartheta}{2}\right)}$$

- Small angular deflections are more frequent (guess why)
- The incident tracks undergo multiple scatterings, each with small angular deflections
- A full description of the Multiple Coulomb Scattering (MCS) cumulative effect on the lateral beam spread is given by the Moliére theory.
- For small angles and thin materials, the net angular deflection can be approximated by a Gaussian with a width $\sigma\theta$ derived by Highland

Multiple Coulomb Scattering (MCS)



$$f(\vartheta) = \frac{1}{\sqrt{2\pi\vartheta_0}} \exp\left(-\frac{\vartheta^2}{2\vartheta_0^2}\right) \qquad \vartheta_0 = \frac{13.6MeV}{\beta cp} z \sqrt{\frac{\Delta x}{X_0}} \left(1 + 0.038 \ln\left(\frac{\Delta x}{X_0}\right)\right)$$

• where p and β are the particle momentum and velocity respectively, Δx and X0 are the thickness and the radiation length of the absorbing material

$$X_0 = 716.4 ext{ g cm}^{-2} rac{A}{Z(Z+1) \ln rac{287}{\sqrt{Z}}} \qquad \qquad X_0 \sim 1/z^2,$$

- Values of X0 for common materials can be calculated and they are present in different databases
- Given the same material thickness, large Z material causes more scatterings

Questions: at PT energies and at the same depth, the MCS is more relevant for protons or ¹²C ions?



Multiple Coulomb Scattering (MCS)



Lateral spread of photon, proton and carbon ion beams as a function of the penetration depth



- Given the same particle range (different particle energies), low Z projectile causes more scatterings
- **Highland approx. fails at large angles** due to nuclear interactions that can produce a large angle deviation even in a single collision
- Highland approx is valid only for thin materials. The particle velocity changes as a function of its position in the target and the β p term at the denominator is not constant

MCS: lateral beam spread



MCS: lateral beam spread



There are two main nuclear interactions:

- **Elastic collisions** where the kinetic energy is conserved and it contributes to the lateral beam spread above all at large angles
- Inelastic collisions where only the total momentum is conserved and it could lead to the production of secondary particles. A specific inelastic reaction has usually an energy threshold below which it cannot occur
- Nuclear inelastic interaction is a many-body problem and the fundamental theory that describes this phenomenon is the quantum chromodynamics (QCD), which application with a non perturbation theory is not feasible for the energy of interest of PT
- At the moment, there are different semi-empirical models developed to describe the data
- Commonly, the nuclear inelastic interactions can be described as a two stage process

- In the first dynamic abrasion stage, which occurs with a time scale of about 10^{-22-23} s
- the projectile interacts with the overlapping nucleons of the target nucleus.
- The result is the formation of an excited projectile pre-fragment with almost the same initial velocity, direction and the ratio of mass over nuclear charge of the incident particle
- $((A/Z)f \sim (A/Z)i)$
- There is an isotropic production of light particles and a slowly recoiled quasi-target fragment.
- In proton therapy the projectile cannot fragment. Thus, the abrasion process leads only to the production of fragments derived from the target nuclei

The following abrasion stage occurs with a time scale of about $10^{-18} - 10^{-16}$ s. It consists in the de-excitation of the fragments and the light nuclei mainly by means of nuclear evaporation, leading to the production of γ -rays, protons, neutrons and light fragments with a kinetic energy of few MeV

- Nuclear evaporation: emission of light fragments ($Z \le 2$) with a kinetic energy of few MeV, similar to the evaporation of a hot system.
- Fermi break-up: the nucleus breaks into lighter fragments if the excitation energy exceeds the binding energy of the fragmentation channels. This effect occurs only for light nuclei with A ≤ 16, which is the typical scenario in PT.
- Fission: The heavy ($Z \ge 65$) excited nucleus can break into two fragments. Since in the human body such heavy nuclei are not present, this is not relevant for PT.
- Gamma emission: The last stage of the de-excitation process is given by the emission of γ rays in order to reach a final configuration only with stable nuclei



Very simple nuclear interaction model

 A simple initial method to evaluate the reaction cross section σR is to adopt a geometric approximation in which the nucleus is assumed to be a "black" sphere with radius a:

 $\sigma R = \sigma T - \sigma e I = \pi (A_p + A_T)^2$

where σT and σel are the total and the elastic cross section, and Ap and AT are the projectile and the target nucleus radius

- Different models has been developed to parametrize the nuclear reactions:
- $\sigma R(E) = \pi r_0^2 (A_P^{1/3} + A^{1/3} b)^2$

where r_0 is the nucleon radius, A is the number of nucleons and b a correction factor. This is the Bradt-Peters formula and it is a good approximation only for particles at very high energy (> 1.5 GeV/u), not suitable for PT applications.

Very simple nuclear interaction model

• $\sigma R(E) = \pi r_0^2 C_1(E) (A_P^{1/3} + A^{1/3} - C_2(E))^2$

in which c1(E) and c2(E) are energy dependant parameters. This parametrization is exploited in the NASA transport code HZETRN for cosmic radiation both for heavy and light ions

• $\sigma R(E) = \pi r_0^2 (1 + A_P^{1/3} - b_0 (1 + A_T^{-1/3})^2 f(E, Z_T))$

where ZT is the charge of the target nucleus, $f(E, Z_T)$ is an energy and target dependant function relevant at low energy (E < 200 MeV) and b0 is the transparency parameter of the Bradt-Peters formula, which can be considered as an expansion of the target nucleons number: b0 = 2.247 - 0.915(1 + $A_T^{-1/3}$).

This formula is adopted to describe the proton-nucleus interactions in the HIBRAC code that has been developed specifically for PT applications.

At energy below 200 MeV, the function $f(E, Z_T)$ is required to include the cross section enhancement. It has dif- ferent shape and parametrization depending on the projectile charge and energy range

Very simple nuclear interaction model



Questions: Do I need to know all the nuclear physics models and parameters? How to evaluate the nuclear interaction effects more "easily"?

FLUKA Monte Carlo models of interest for PT



Electromagnetic interactions models in FLUKA

Handron-nucleus interactions:

- PreEquilibrium Approach to NUclear Thermalization (PEANUT) model for particles with P<3-5 GeV/c based on Generalized Intra-Nuclear Cascade (GINC) model
- Pre-equilibrium emission of light nuclei (A<5)
- Evaporation, Fission, Fragmentation and y de-excitation

Nucleus-nucleus interactions

- Boltzmann-Master Equation model (E<100 MeV/u): Thermalization of composite nuclei by means of two-body interactions and secondary particles emissions Cavinato et al, Nuclear Physics A 643 (1998)
- **Relativistic Quantum Molecular Dynamics** (0.12-5 GeV/u): Collision simulated minimizing the Hamiltonian equation of motion considering the Gaussian wave functions of all the nucleons in the nucleus overlapping region

H. Sorge et al., Annals of Phys. 192 (1989) 266

Monte Carlo simulation tools



- There are different Monte Carlo simulation tools adopted not only in PT, but in general in physics
- Each MC tool has its advantages and contraries
- Each MC tool can be used in a specific range of projectile, target etc.
- Two of the most "famous" MC simulation tools are GEANT4 and FLUKA since they can be used in a wide range of experimental context
- For specific uses, other MC tools could be more suitable
- However, all the MC outcome should be benchmarked with experimental data

Nuclear inelastic interactions in hadrontherapy

Fragment	E (MeV)	LET (keV/um)	Range (um)
¹⁵ O	1.0	983	2.3
¹⁵ N	1.0	925	2.5
^{14}N	2.0	1137	3.6
¹³ C	3.0	951	5.4
¹² C	3.8	912	6.2
¹¹ C	4.6	878	7.0
^{10}B	5.4	643	9.9
⁸ Be	6.4	400	15.7
⁶ Li	6.8	215	26.7
⁴ He	6.0	77	48.5
³ He	4.7	89	38.8
^{2}H	2.5	14	68.9

Figure 2.1: Expected average physical parameters for target fragments produced in water by a 180 MeV proton beam [32].



build-up of secondary fragments produced by 400 MeV/u 12C ions stopped in water $signod b = 10^{4} - \frac{10^{4}}{10^{2}} + \frac{10^{4}}{10^{2}} + \frac{10^{4}}{10^{2}} + \frac{10^{4}}{10^{4}} + \frac{10$

Angular distribution of fragments produced by an 16O@200 MeV/u on a 2 mm target of C2H4, Fluka sim Mainly two effects:

Target fragmentation



• **Projectile fragmentation** in heavy ion therapy (why not in protontherapy?)



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Depth-dose profile





Bragg curve



Non-elastic nuclear reactions move dose from the peak upstream.

Pencil Beam

Bragg curve summary



- The increase of dE/dx as the charged particle slow down causes the overall \ upwards sweep
- The depth of penetration increases with beam energy
- The width of the peak is due to the range straggling and beam energy spread
- The overall shape depends on the beam's transverse size
- Inelastic nuclear reactions move dose from the peak upstream
- A short effective source distance reduces the peak/entrance ratio
- Low energy beam contamination (e.g.: from collimator scatter) can affect the entrance region

Questions

Photon attenuation coefficient



1) very narrow photon beam

2) thin attenuator (small number of interaction events)

3) small detector and far away from the attenuator \rightarrow The only photons that we want to measure are those that **undergo no interaction**

The number of photons (n) which interact in the attenuator is proportional to the number of incident photons (N) and the thickness of the attenuator (Δx) .

The constant of proportionality $\boldsymbol{\mu}$ is the linear attenuation coefficient

Valid for *n* << *N*, i.e. **thin attenuator**

- Do you remember this slide stolen from Prof. Veronese lecture 4?
- Well, this is valid for photons, what about heavy charged particles?
- What are the interactions in case of heavy charged particles?
- How can we define a "good geometry" in case of heavy charged particles?

References for this lesson

• Prof. Veronese's lessons

The basic concepts presented here are really "basic", so you can find tons of suitable material almost everywhere. However, here a couple of useful books for particle interactions and detectors:

- "Radiation Detection and Measurement", Glenn F. Knoll
- "Techniques for nuclear and particle physics experiments", W. R. Leo
- Ask me if you need some of the material!



Pint of science



